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Modelling uncertainty induced by plastic hinge length in lumped-plasticity analysis of RC columns

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ABSTRACT

This study quantifies the influence of various plastic hinge length assumptions on the uncertainty in the response of reinforced concrete (RC) columns simulated using lumped plasticity models. Although several expressions have been proposed in the literature to estimate the plastic hinge lengths of RC members, the plastic hinge length values computed using these equations vary considerably and only a limited number of studies have attempted to quantify the effect of using different plastic hinge lengths on the simulated response of RC members. In this study, concentrated plasticity models of RC columns tests reported in literature are developed using Opensees. Different equations proposed in the literature to estimate plastic hinge length are used in conjunction with section (moment-curvature) analysis to generate the inelastic moment-rotation backbone of the plastic hinges. The uncertainty associated with plastic hinge length estimation on the simulated member response is quantified by conducting nonlinear static analyses. Further, fibre-element modelling of the selected RC columns is carried out and the results are compared with the results from plastic-hinge models to understand where the distributed plasticity prediction sits within the range of lumped-plasticity predictions.

1 INTRODUCTION

Static pushover analysis is commonly used in engineering practice when predicting seismic force and deformation demands of structures. Modelling is a key step in the application of pushover analysis, and the adopted model must incorporate the nonlinear behaviour of the structure (or materials). Lumped plasticity idealization is a commonly used modelling technique to estimate deformation capacities. The nonlinear deformation capacity of an element depends on its ultimate curvature and plastic hinge length (Krawinkler 1996).

Plastic hinge length is the portion of the member length, where moment demand at the ultimate limit state exceeds the yielding capacity (Megalooikonomou et al. 2017). In lumped-plasticity modelling of structures, an estimate of the plastic hinge length is used to determine the moment-rotation backbone of a rotational plastic hinge from the moment-curvature relationship of the section. Therefore, accurate assessment of plastic hinge length is important to estimate the response of structural systems with reasonable accuracy. As a result, several researchers have proposed a number of methods for estimating the plastic hinge length of reinforced concrete

(RC) members. The plastic hinge lengths obtained for the same RC member using the different expressions vary over a wide range.

Inel and Ozman (2006) compared the deformation capacity of RC concrete frames using different plastic hinge lengths and concluded that plastic hinge length has a significant impact on the displacement capacity of the RC frames. According to their comparisons, the displacement capacities can vary by approximately 30% based on the plastic hinge length expressions used. Although the study highlighted the significant uncertainties induced by using different plastic hinge length, it did not systematically evaluate the performance of different expressions in the predicted structural response.

Therefore, in this paper, different expressions to estimate the plastic hinge length of RC structures are summarised and variation in the plastic hinge length of a few RC columns as a result of different approximation methods is quantified. Further, the columns are modelled using two different modelling approaches (distributed-plasticity; i.e. fibre-element, and lumped-plasticity; i.e. rotational spring) and the uncertainty associated with the prediction of column deformation capacity as a result of different plastic hinge lengths is quantified.

2 PLASTIC HINGE LENGTH EXPRESSIONS PROPOSED IN LITERATURE

Ten plastic hinge length estimation equations are summarised from the literature in Table 2 below. It is shown that the majority of the plastic hinge estimation expressions contain column section depth along the bending direction and column height. These terms mainly account for the column bending. Some expressions also include the longitudinal diameter and yield strength of bars, which are intended to account for bar slip due to the elongation of longitudinal bars (Priestley et al. 1987).

Table 1: Empirical expressions of plastic hinge length.

Researcher/s and/or Reference	Plastic hinge length expression (mm)
Japanese guidelines (2003)	$\mathrm{Min}\ (0.2\mathrm{L} - 0.1\mathrm{D}, 0.5\mathrm{D})$
Priestley and Park (1987)	$0.08L + 6d_s$
Sawyer (1964)	$0.25D + 0.075L$
Eurocode 8 (2005)	$0.1L + 0.015f_y d_s$
Paulay and Priestley (1992)	$0.08L + 0.022f_y d_s$
Chinese guidelines (2008)	$Min\ (0.08L + 0.022f_y d_s, 0.667D)$
AASHTO (2002)	$0.08L + 9d_s$
New Zealand standard (NZS 3101:2006)	$0.5D$
Corley (1966)	$0.5D + 0.2 * \frac{L}{\sqrt{D}}\ \ (inch)$
Mattock (1967)	$0.5D + 0.05L$

Note: L = length from the point of maximum moment to the point of inflection; D = column section depth along the bending direction; d_s = longitudinal bar diameter; f_y = longitudinal bar yield strength

In order to quantify the influence of plastic hinge length estimation on the simulated response of RC members, ten previously tested RC columns with a wide range of properties (axial load ratio, reinforcement ratio and material properties) are selected from the PEER column database (Pacific Earthquake Engineering Research Center, 2003). These columns are then modelled in OpenSees using the fibre-element and lumped-plasticity modelling techniques. The properties of the tested specimens are summarised in Table 1 below.

Table 2: Summary of test columns properties.

		f_c (MPa)	f_y (MPa)	D (mm)	B (mm)	L (mm)	ρ_l	ALR
1	Tanaka and Park 1990, No. 7	32	511	550	550	1650	0.0125	0.301
2	Saatcioglu and Ozcebe 1989, U3	34.8	430	350	350	1000	0.0321	0.141
3	Saatcioglu and Grira 1999, BG-1	34	455.6	350	350	1645	0.0195	0.428
4	Saatcioglu and Grira 1999, BG-4	34	455.6	350	350	1645	0.0293	0.462
5	Saatcioglu and Grira 1999, BG-8	34	455.6	350	350	1645	0.0293	0.231
6	Matamoros et al. 1999, C10-20N	65.5	572.3	203	203	610	0.0193	0.211
7	Mo and Wang 2000, C1-1	24.9	497	400	400	1400	0.0213	0.113
8	Bechtoula, Kono, Arai and Watanabe, 2002, D1N30	37.6	461	250	250	625	0.0243	0.300
9	Bechtoula, Kono, Arai and Watanabe, 2002, L1N6B	32.2	388	560	560	1200	0.0194	0.594
10	Mo and Wang 2000, C3-3	26.9	497	400	400	1400	0.0213	0.209

Note: f_c = unconfined concrete strength; f_y = strength of longitudinal reinforcing bar; D = column section depth along the bending direction; B = column section width across the bending direction; L = column height; ρ_l = longitudinal reinforcement ratio; ALR = axial load ratio

Figure 1(a) below shows the accumulated probability of exceedance of plastic hinge length for each column. For simplification, the plastic hinge length in Figure 1 is normalised with respect to the column section depth. The figure shows that due to the dependency of these plastic hinge equations on other column parameters, the normalised plastic hinge length varied between different columns. Figure 1(b) shows the variability of each plastic hinge length equation for all the columns. It is noted that plastic hinge lengths calculated from New Zealand standard stay constant as a proportion of column depth (along the bending direction) since plastic hinge length equation from the New Zealand standard only contains column depth.

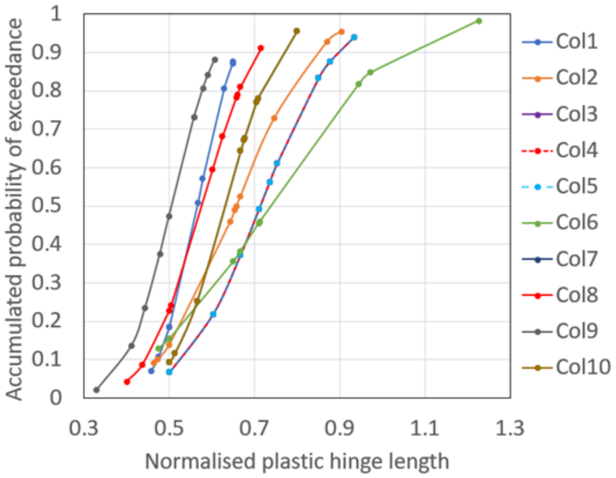


Figure 1(a): The accumulated probability of exceedance of normalised plastic hinge length for each column

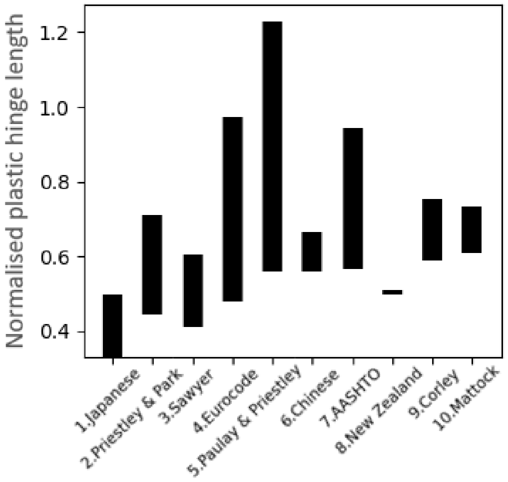


Figure 1(b): The variability of each plastic hinge length equation for all the columns

Figure 2 below is a fragility curve that shows the accumulated probability of exceedance of the normalised plastic hinge length for all the columns. This figure clearly indicates that the plastic hinge length varied quite a bit for each expression. Plastic hinge length prediction using the Japanese guidelines, Sawyer, New Zealand Standard, Corley and Mattock, are mainly related to the column depth and height, and therefore, the variability of plastic hinge length between columns is relatively low for these expressions. Whereas, in addition to column depth and length the other expressions also incorporate the effect of yield strength of longitudinal bar and bar diameter on the plastic hinge length of RC members. As a result, the variability of plastic hinge length estimated using these expressions is high. The mean value of the normalised plastic hinge length is 0.644 and the coefficient of variation of the function is 23.6%.

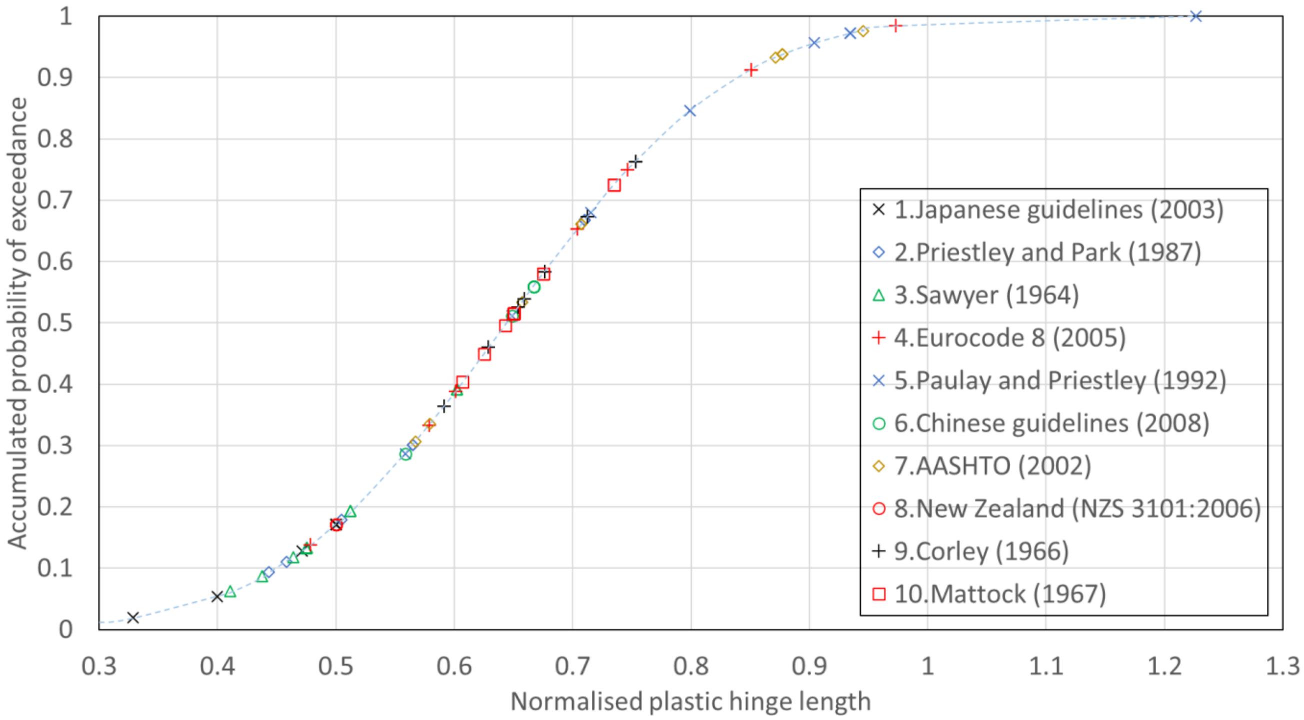


Figure 2: Cumulative lognormal distribution of normalised plastic hinge lengths

3 MODEL DEVELOPMENT

In order to quantify the effect of using different plastic hinge lengths on the simulated response of RC members, fibre element and lumped plasticity models of RC columns are developed in OpenSees.

3.1 Fibre element model

The fibre element RC column is developed by discretising each column into six displacement-based fibre elements connected end-to-end. The length of the bottom element is calibrated to account for the strain-localisation effect (Dhakal, 2000). The remaining five elements are then evenly distributed over the height of the column. Each element is further discretised into five fibre sections containing confined concrete, unconfined concrete and steel fibres (as shown in Figure 3).

The monotonic behaviour of concrete is simulated by the Concrete02 material model in OpenSees. The stress-strain characteristics of confined concrete adopt the concrete model proposed by Saatcioglu and Razvi (1992). The hysteretic model in OpenSees is used to simulate the behaviour of reinforcing bars. The tension and compression response envelopes are defined using the bar buckling model proposed by Dhakal and Maekawa (2002). The tension envelope consists of three parts: an elastic branch, a yield plateau and a strain-hardening branch. The compression behaviour of the model is defined by introducing a non-dimensional buckling parameter (λ) as:

$$\lambda = \frac{L}{D}\sqrt{f_y} \tag{1}$$

Where slenderness ratio (L/D) is the ratio of the buckling length (i.e. integral multiple of the transverse reinforcement spacing) (L) to the diameter (D) of the bar.

3.2 Lumped plasticity model

For the lumped plasticity model, two nodes are overlapped at the bottom of the RC column. A zero-length element that acts as a rotational spring is then modelled to connect these two nodes. Nonlinear section analysis is carried out using OpenSees to obtain the moment curvature response of the RC sections. Thereafter, bi-linearisation of the moment-curvature curve is carried out to extract the yield curvature ($\emptyset_y$), yield moment (M_y), capping curvature ($\emptyset_c$) and capping moment (M_c) for the rotational spring. Yield curvature and capping curvature of the section is then transformed to moment rotation response as:

$$\theta_y = \frac{\emptyset_y L}{3} \tag{2}$$

$$\theta_c = (\emptyset_c - \emptyset_y)l_p + \theta_y \tag{3}$$

Where L is the height of the column; l_p is plastic hinge length; θ_y is yield rotation of the spring; θ_c is capping rotation of the spring.

In lumped-plasticity modelling, the flexural member is modelled by connecting the rotational spring and elastic beam-column elements in series. Therefore, the stiffness of rotational spring and elastic elements need to be modified so that the overall stiffness equals the rotational stiffness of the member (Ibarra, 2004).

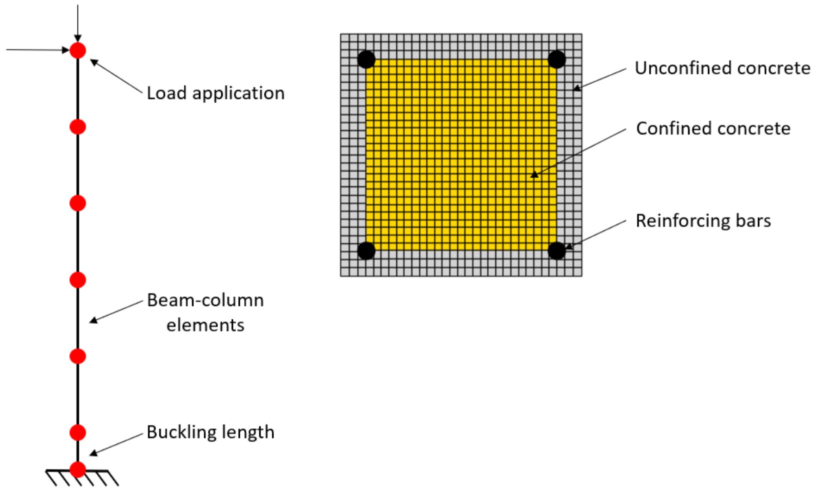


Figure 3: Fibre element model layout

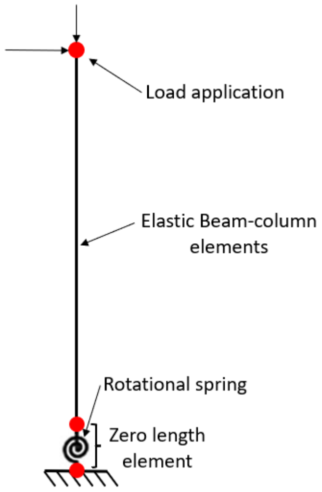


Figure 4: Lumped-plasticity model layout

4 PUSHOVER ANALYSIS

In order to verify the performance of the fibre element model, cyclic pushover analysis is carried out for columns using fibre element model. Figure 5 shows the comparison of the experimental and analytically-simulated stress-strain responses of a tested column (column No.5 in Table 2) subjected to cyclic loading. As shown in the figure, the fibre element model is capable of simulating the cyclic response of the tested column with reasonable accuracy.

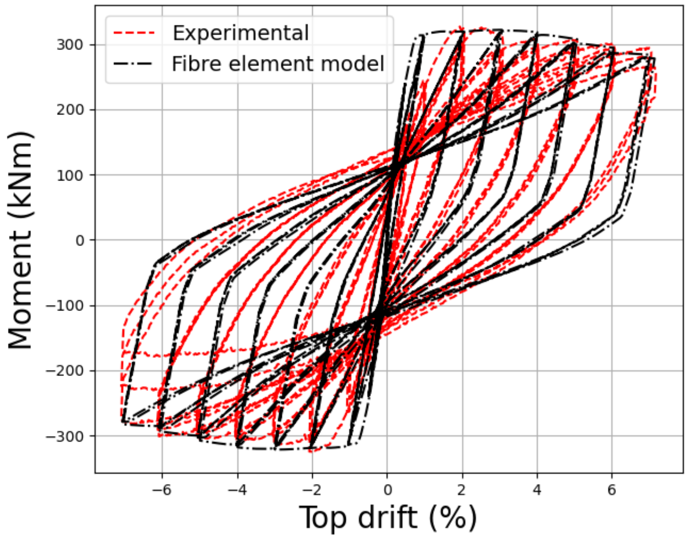
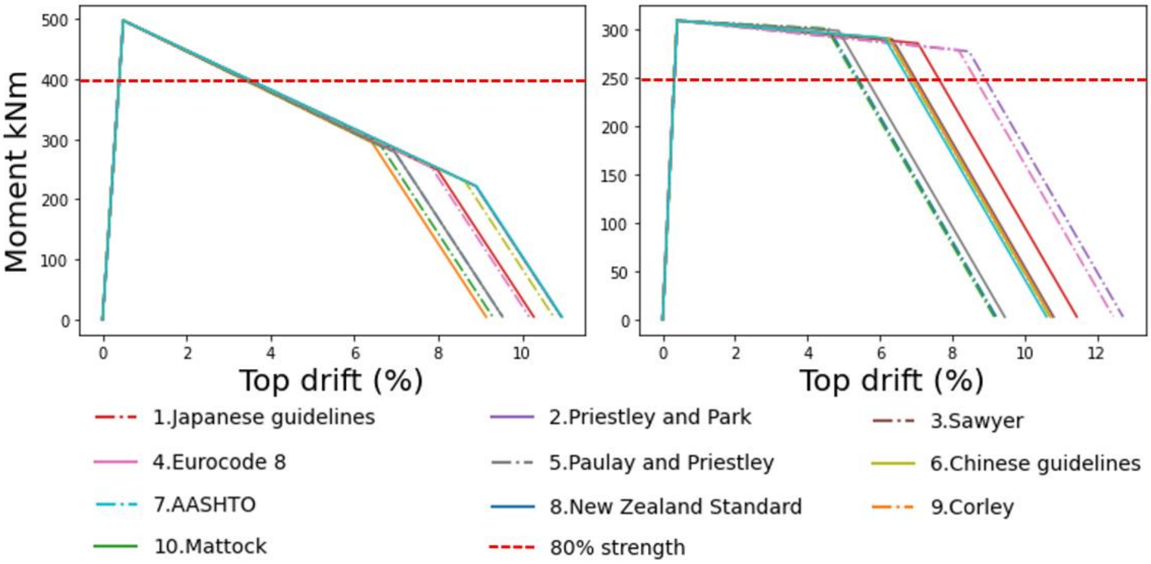


Figure 5: Comparison of hysteresis response of the tested column with fibre element model prediction

After the verification, monotonic pushover analysis is carried out for all the columns using both fibre element and lumped-plasticity models. the failure drifts are compared between the two analytical models. In this study, the failure point is defined as the point where column strength drops below 80% of its maximum strength.

5 RESULTS

The results from pushover analysis can be categorised into two groups depending upon the failure drift, i.e. the failure criteria is met before or after the capping drift is reached (where the plastic hinge strain-hardening phase ends). The red line in Figure 6 indicates 80% of its maximum strength. Figure 6a shows the case where the failure happens before the capping drift is achieved. This was usually observed in the columns subjected to a high axial load ratio (axial load ratio higher than 20 percent). In this case, the difference in the top drift at failure between the expressions is small. Figure 6b shows the case when the column fails after capping. This occurs when the column is subjected to a low axial load ratio (axial load ratio lower than 20 percent). And the difference in the top drift at failure is significant between expressions.



(a) Group 1: Failure before capping *(b) Group 2: Failure after capping*

Figure 6: Lumped-plasticity model pushover curve

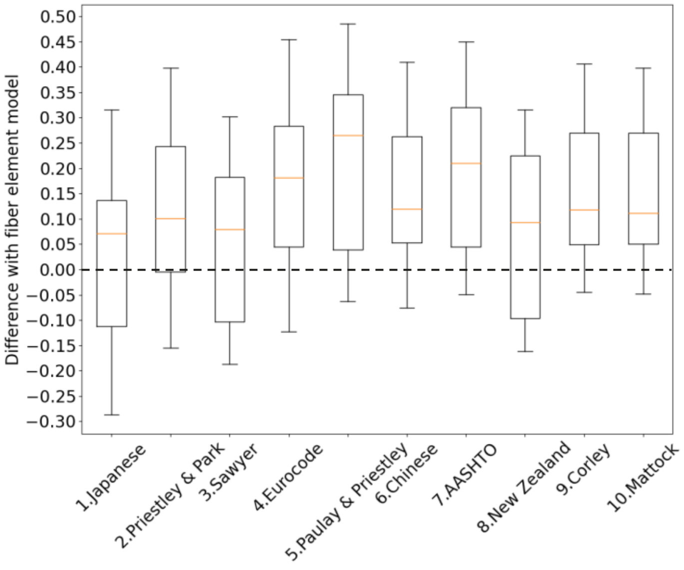


Figure 7: Difference of failure drift between fibre model and lumped-plasticity model

Figure 7 above is a box plot that summarises the difference of failure drift between the fibre-element model and lumped plasticity model with different plastic hinge length. In this box plot, the orange line indicates the median value of failure drift for each expression, the vertical line indicates the range of the dataset and the box in the middle indicates the range between the median of the upper and lower half of the dataset. A positive value indicates that the failure drift estimated using the lumped-plasticity modelling is higher than that of the failure drift obtained from fibre-element modelling. The box plot clearly shows that regardless of the chosen plastic-hinge approximation method the plastic-hinge models tend to overestimate the failure drift as compared to fibre-element models. Further, among all the plastic-hinge expressions identified in this study, the use of expression proposed by the Japanese guidelines tends to provide the most conservative estimate of the drift capacity. This is mainly attributed to the fact that Japanese guideline tends to give a lower-bound estimate of the plastic-hinge length (as can be noticed in Figure 2).

Figure 8 shows the mean squared difference (MSD) of drift capacity between the fibre element model and lumped plasticity model. Herein, MSD is the average squared difference between fibre model and lumped plasticity model results in drift capacity. MSD incorporates both the variance of the estimator (how widely spread the estimates are from one data sample to another) and its bias (how far off the average estimated value is from the reference value). It shows that the expression proposed by Sawyer gives the lowest MSD in drift capacity, and the expression proposed by Paulay and Priestley gives the largest MSD in drift capacity compared to the fibre element model. By comparing the MSD results with the plastic hinge expressions (Table 1), it is found that plastic hinge estimation expressions that contain yield strength and diameter of longitudinal bar (to account for bar slip) have higher MSD. This is mainly because the fibre element model does not incorporate the bar slip effect.

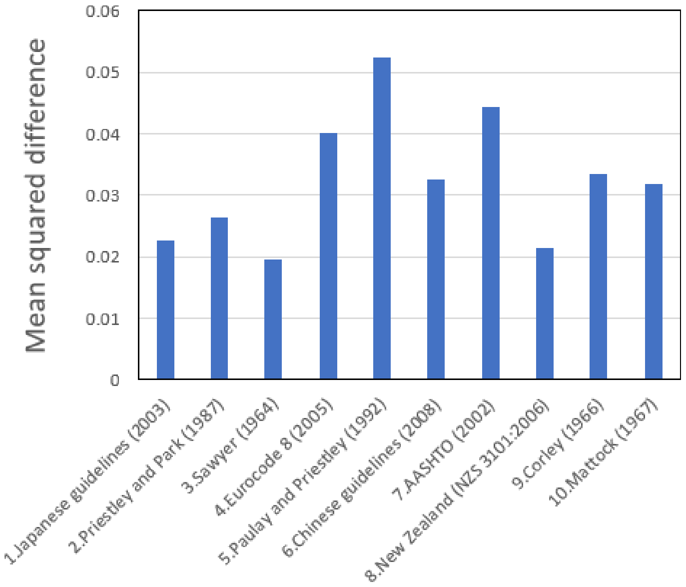


Figure 8: Mean squared difference of drift capacity between fibre element and lumped plasticity model

6 DISCUSSION

In order to investigate how the difference in plastic hinge length propagates to drift capacity, the box plot (Figure 7) is compared to the fragility curve (Figure 2) and the MSD plot (Figure 8).

1. Plastic-hinge length expressions proposed by Japanese guideline, Sawyer and New Zealand Standards contain only the column depth and height in the equations. Plastic hinge lengths calculated from these expressions are distributed over a limited range and provide a lower-bound estimate of the plastic-hinge length. And as a result, the three expressions above have a relatively conservative estimation in drift capacity and a lower MSD than other expressions (median value closer to zero).
2. Expressions proposed by Chinese guideline, Corley and Mattock have similar drift capacity and median values as shown in the box plot. By comparing this to Figure 2, it is found that the plastic hinge length calculated using these expressions provides a median estimate of the plastic-hinge length.
3. Expressions proposed by Priestley and Park, Eurocode8 and AASHTO resulted in a similar drift capacity for the test columns. It can be seen from the fragility curve shown in Figure 2, the plastic

hinge length calculated from these expressions provide an upper-bound estimate for most of the analysed columns. This also explains why the above expressions have the most significant MSD in figure 8.

7 CONCLUSIONS

In this paper, ten plastic hinge length expressions were summarised from the literature and a fragility curve that shows the cumulative probability of exceedance of plastic hinge lengths for different columns is developed. Fibre element and lumped-plasticity models were developed for the selected cantilever columns from the PEER column database, and pushover analyses were undertaken. Drift capacities of the identified columns were estimated using the two modelling approaches and the influence of uncertainty associated with estimating plastic hinge lengths on the simulated deformation capacity of RC columns was estimated. The key conclusions drawn from this study are:

1. The mean value of the normalised plastic hinge length is 0.644 and the coefficient of variation of it is 23.6%.
2. The plastic hinge modelling tends to consistently overestimate the drift of RC columns as compared to the results estimated using fibre element modelling.
3. Plastic hinge length estimation expressions that contain yield strength and diameter of longitudinal bar tend to give a higher MSD in drift capacities compared with the fibre element model result.
4. Within the different plastic hinge models used in this study, the drift capacity estimated using the plastic-hinge models based on the plastic hinge length prediction using the Japanese guideline gives the most conservative results.
5. A plastic hinge length in the lower-bound of the fragility curve gives a closer drift capacity compared with the fibre element model.

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