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ROBUST AND EFFICIENT NONLINEAR STRUCTURAL ANALYSIS USING THE CENTRAL DIFFERENCE TIME INTEGRATION SCHEME

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Abstract. *The explicit central difference numerical time integration scheme is demonstrated to be a robust and efficient alternative to commonly used implicit schemes like the Newmark average acceleration scheme for nonlinear structural response simulation. Numerical non-convergence issues, which are frequently encountered using the Newmark average acceleration scheme, are shown to introduce conservative biases in the estimated structural capacity and hamper the efficiency of analysis. They are shown to be responsible for the underestimation of the median collapse capacity of a 9-storey steel moment frame building by 10%. Despite requiring shorter analysis time steps, the time taken to conduct an incremental dynamic analysis using the central difference scheme is 73% lower than using the average acceleration scheme.*

1 INTRODUCTION

Nonlinear response history analysis has witnessed increased adoption in seismic design and assessment practice in recent years [1], [2], especially for the design and assessment of tall and important structures. In current research and practice, nonlinear structural response simulations are almost exclusively conducted using implicit numerical time integration schemes, of which, the Newmark average acceleration scheme is the most popular. Implicit schemes are, however, inherently iterative in nature and often fail to converge to a solution when used to simulate the response of structures under intense earthquake ground motions, at or close to their ultimate collapse limit states. The use of implicit schemes, therefore, possesses the potential to introduce conservative biases in the estimated structural capacity. Nevertheless, numerical issues like non-convergence typically receive little attention in comparison to structural modelling and ground motion selection considerations. This study proposes the use of the explicit central difference time integration scheme as a robust and efficient alternative to the Newmark average acceleration scheme for nonlinear structural analysis.

2 NEWMARK AVERAGE ACCELERATION SCHEME

The Newmark average acceleration scheme is the most widely used numerical time integration scheme for response history analysis. The reason for its popularity is its unconditionally stable nature, which permits the use of relatively large analysis time steps. Being an implicit scheme, it enforces equilibrium at the end of each time step which makes it inherently iterative in nature, and convergence of the iterations is not guaranteed. Numerical non-convergence is, in fact, a frequently encountered phenomenon, and the likelihood of encountering non-convergence increases when analysing complex structural models under long and intense ground motions. Upon encountering numerical non-convergence, a series of workarounds is typically employed to overcome it, including trying different solution algorithms and other time integration schemes, decreasing the analysis time step, and raising the convergence tolerance [3]–[5]. These strategies are not always successful, but are computationally intensive and can hamper the efficiency of the analysis if invoked too frequently. If all attempts fail, it is common practice to declare structural collapse, although often incorrectly as demonstrated in this study and others (e.g., [4], [6]).

3 CENTRAL DIFFERENCE SCHEME

The central difference scheme is an explicit scheme since it enforces equilibrium at the beginning of each time step. Being non-iterative in nature, it effectively sidesteps the issue of numerical non-convergence. This makes it a popular choice when conducting simulations that involve large nonlinear deformations like blast and crash simulations [7], [8]. Structural response simulations that involve large nonlinear deformations are, therefore, also expected to benefit from using the central difference scheme, despite the longer duration of earthquake loads when compared to impulse loads like blast and crash loads.

The most commonly cited drawback of the central difference scheme is its conditionally stable nature, which limits the largest analysis time step it can be used with to $\Delta t_{max} = T_{min}/\pi$, where T_{min} is the shortest modal period. This condition precludes the presence of any massless degrees of freedom and extremely stiff elements or penalty constraints in the structural model, which entails some additional effort during model creation. These requirements are, however, not unique to the central difference scheme since they have also been linked to improved convergence performance of implicit schemes [4], [9, Sec. 9.5.2].

The dynamic tangent matrix that needs to be factorised at each time step when using the central difference scheme is a linear combination of just the mass and damping matrices. Since the mass matrix is typically constant, using a constant damping matrix (like a modal damping matrix) would require the dynamic tangent matrix to be factorised only once during the entire simulation, thereby vastly improving the efficiency of the scheme. When a simulation is conducted in parallel by domain decomposition, this would also significantly minimise the communication overhead between the processors. Since the duration of each analysis is known to be proportional to the length of the accelerogram, it also permits the use of efficient static parallel load balancing techniques when analyzing response under different ground motions on different processors.

4 COMPARISON OF ROBUSTNESS

A concentrated plastic hinge model of the 9-storey steel moment frame building designed as part of the SAC Steel Project [10], was created in OpenSees. The hysteretic behaviour of the plastic hinges was modelled using the Ibarra-Medina-Krawinkler bilinear hysteretic model [11], and the destabilising $P - \Delta$ effect of the adjacent gravity frame was captured using a pin-connected leaning column. The collapse capacity of the frame was estimated separately using the Newmark average acceleration and the central difference time integration schemes by conducting incremental dynamic analysis (IDA) [12] using 44 ground motions from the FEMA P695 [13] far-field record set.

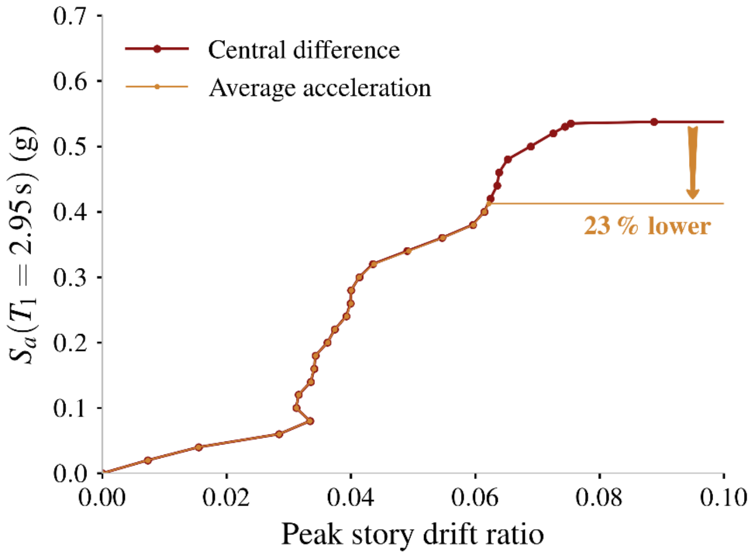

Figure 1: Comparison of the IDA curves computed using the central difference and Newmark average acceleration time integration schemes for one of the 44 ground motions

Upon encountering numerical non-convergence using the average acceleration scheme, the following sequence of efforts were made to overcome it: (i) different solution algorithms were tried; (ii) the analysis time step was sequentially decreased; and (iii) other implicit time-integration schemes (including some with algorithmic damping like the HHT-α scheme) were used. If all efforts failed, structural collapse was declared as per conventional practice. For 12 out of the 44 ground motions, the collapse intensity obtained using the average acceleration scheme was found to be lower than that estimated using the central difference scheme by 10% or more. The IDA curves for one of these ground motions is plotted in Figure 1. The IDA curves computed using the two schemes are seen to be identical until they bifurcate at a certain intensity level. This corresponds to the intensity level at which persistent numerical non-convergence was encountered using the average acceleration scheme, prompting the premature declaration of structural collapse. The central difference scheme, on the other hand, was able to successfully evaluate the response at that intensity level and a few others above it. The difference between the collapse intensities estimated using the two schemes was lower than 1% for 29 out of the 44 ground motions. The net effect of the premature declaration of collapse for 12 of the 44 ground motions was an underestimation of the median collapse capacity by 10% using the average acceleration scheme. This clearly demonstrate the robust nature of the central difference scheme, which can be attributed to its immunity against numerical non-convergence.

5 COMPARISON OF EFFICIENCY

The simulations were conducted using a time step of 5×10^{-3} s using the average acceleration scheme, and 1.5×10^{-4} s using the central difference scheme. The time taken to analyse the structure under the ground motion recorded during the 1992 Landers earthquake at the Coolwater station using the average acceleration scheme was 1.0 min when it was scaled to a low scale factor where non-convergence was not encountered. The runtime, however, increased to 20.9 min at a higher scale factor where numerical non-convergence compelled the use of computationally intensive strategies to overcome it. The runtime using the central difference scheme was only 3.3 min using a constant damping matrix that required only a single factorisation. Thus we see that although the central difference scheme requires the use of a smaller analysis time step, the system of equations can be solved more efficiently at each time step.

The time taken to conduct the entire IDA in parallel on 160 processors using dynamic load balancing was 118 min using the average acceleration scheme and 32 min using the central difference scheme. The 73% shorter runtime using the central difference scheme can be attributed to the large number of instances where numerical non-convergence was encountered using the average acceleration scheme, forcing the use of computationally intensive steps to overcome it. Thus, the central difference scheme is seen to be a competitive alternative to the Newmark average acceleration scheme, not just in terms of robustness, but also in terms of efficiency. These findings are consistent with other previous studies like [14]–[16].

6 CONCLUSION

The central difference time integration scheme is demonstrated to be a robust and efficient alternative to the Newmark average acceleration scheme for nonlinear structural analysis. Its robustness is attributed to its non-iterative nature, which renders it immune to numerical non-convergence issues. Numerical non-convergence was shown to be responsible for the underestimation of the median collapse capacity of a 9-storey steel moment frame building by 10% when using the average acceleration scheme. The time taken to conduct IDA using 44 ground motions was shown to be 73% lower using the central difference scheme due to the large number of instances numerical non-convergence was encountered using the average acceleration scheme, thereby necessitating the use of computationally intensive workarounds to overcome it.

The only drawback of the central difference scheme is its conditionally stable nature which imposes restrictions on the analysis time step and entails additional effort during model creation to assign mass (or moment of inertia) to all degrees of freedom and avoid the use of stiff elements or penalty constraints.

With the inevitable gradual shift towards more complex structural models and statistically rigorous analysis procedures involving large numbers of ground motions and structural model realisations, it is imperative that adequate attention is paid to the accuracy and efficiency of the numerical solution strategies employed to conduct the simulations.

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