

# **Influence of Ground Motion Duration on the Collapse Response of Bridge Structures**

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## **ABSTRACT**

While it is generally perceived that ground motion duration will influence structural performance, previous research on the topic has produced mixed conclusions, which has led to the effect of duration being largely ignored in structural design practice. The believed reasons for the inconclusive results are the use of non-deteriorating structural models, attention not paid to behavior near collapse and the use of inefficient metrics to characterize duration. This paper summarizes preliminary results of a study that employs non-linear incremental dynamic analyses to assess the effect of ground motion duration on the estimated collapse risk of reinforced concrete bridge piers. Spectrally equivalent long and short duration record sets are used to isolate the effect of duration from that of other ground motion characteristics like response spectral amplitude and response spectral shape, and quantify its influence on estimated seismic collapse risk. Sensitivity of the effect of duration to model parameters is studied to help identify classes of structures most susceptible to long duration shaking. Preliminary findings and their implications on research and structural design practice are presented.

## INTRODUCTION

Although ground motion duration is intuitively expected to influence structural response, a number of previous studies on the subject have produced mixed and inconclusive results (Hancock and Bommer, 2006). The reasons for this are believed to be the following. Firstly, the numerical models employed in previous studies did not generally incorporate cyclic deterioration of strength and stiffness that occurs in most structures and is essential to accurately simulate collapse. Secondly, very few studies studied the effect of ground motion duration on structural collapse and instead focused on structural response under mildly non-linear conditions. Finally, a number of different metrics were used to characterize duration, some of which were not as efficient as others in characterizing the duration of strong shaking as experienced by the structure of interest. As a consequence of the lack of consensus on the effect of ground motion duration on structural performance, current seismic design provisions like ASCE, 2010, performance assessment studies like FEMA, 2009 and loading protocols used for component testing (Krawinkler, 2009) do not explicitly consider the effect of ground motion duration.

The main challenges in studying the effect of duration have been the scarcity of long duration ground motions and the difficulty in isolating the effect of ground motion duration from the effects of other ground motion characteristics like response spectral amplitude, response spectral shape and pulse-like characteristics. However, the number of long duration ground motions recently recorded from the 2008 Wenchuan (China), 2010 Maule (Chile) and 2011 Tohoku (Japan) earthquakes puts us in a better situation today than ever before to study this topic. This study employs a deteriorating non-linear model of a concrete bridge pier and spectrally equivalent long and short duration record sets to demonstrate the effect of ground motion duration on the estimated collapse capacity of structures.

The collapse capacity of a structure is defined as the intensity of ground excitation that causes structural collapse. It is usually treated as a random variable, defined by a lognormal cumulative distribution function called the collapse fragility curve. The first requirement to obtain an accurate estimate of a structure's collapse capacity is a realistic numerical model that accurately characterizes the structure's behavior at small and large deformations. This model must incorporate the expected cyclic and in-cycle deterioration of component strength and stiffness (Ibarra et al., 2005; Lignos and Krawinkler, 2011). A set of ground motions must then be chosen at each intensity level to conduct non-linear dynamic analyses. Since the chosen ground motions form the link between hazard analysis and demand analysis, they must accurately represent the hazard at the site (NIST, 2011). During each analysis, the structure is assumed to have collapsed if an unbounded increase in deformations is observed or if the deformations exceed a rational pre-defined threshold (Haselton and Deierlein, 2007). The probability of collapse at each intensity level is computed as the number of ground motions at that intensity level that caused structural collapse. The collapse fragility curve is then estimated by fitting a lognormal cumulative distribution function through these data points.

As shown in previous studies (e.g., Baker and Cornell, 2006; Baker, 2011), it is important to ensure that the response spectral shapes of the chosen ground motions at each intensity level match the conditional spectrum corresponding to that intensity level. It is demonstrated in this study that in addition to spectral shape, it is important to ensure that the durations of the chosen ground motions match the distribution of expected ground motion durations at the site at each intensity level. Not paying attention to the spectral shapes and durations of the chosen ground motions can lead to erroneous estimates of collapse capacity.

## ANALYSIS OF DURATION METRICS

Before any attempt can be made to analyze the effect of duration on the estimated collapse capacity of a structure, a suitable duration metric must first be chosen to quantify the duration of strong shaking in an accelerogram. The observed correlation between duration and collapse capacity will largely depend on the duration metric employed. A number of definitions of ground motion duration have been used in prior research (Bommer and Martinez-Pereira, 1999), and some are better suited than others for use in hazard characterization and ground motion selection. The following were identified as possible candidates:

- **Bracketed duration** is the time elapsed between the first and last excursions of the accelerogram above a certain acceleration threshold (commonly used thresholds are 0.05g and 0.10g).
- **Significant duration** is the time interval over which a specific percentage of the total energy represented by the integral  $\int a^2 dt$  is accumulated, where  $a$  represents the ground acceleration (commonly used ranges for the accumulated energy are 5% to 95% and 5% to 75%).
- **Arias Intensity** =  $\frac{\pi}{2g} \int_0^{t_{max}} a^2 dt$   
is a measure of the energy contained in an accelerogram, where  $t_{max}$  represents the length of the accelerogram. Although not purely a metric of duration, it involves integration over time and is expected to be correlated to the duration of strong shaking (Kayen and J. K. Mitchell, 1997).
- **Cumulative Absolute Velocity (CAV)** =  $\int_0^{t_{max}} |a| dt$   
is considered for the same reasons as the Arias Intensity above (Kramer and R. A. Mitchell, 2006).
- $I_D = \frac{\int_0^{t_{max}} a^2 dt}{PGA \times PGV}$   
is a dimensionless duration metric proposed by Cosenza and Manfredi, 1997, with  $PGA$  and  $PGV$  representing the peak ground acceleration and peak ground velocity respectively.

Among these, Arias Intensity and Cumulative Absolute Velocity have been used by geotechnical engineers to study the liquefaction potential of soil deposits, since they each implicitly contain information about the amplitude, frequency content and duration of a ground motion (Kayen and J. K. Mitchell, 1997; Kramer and R. A. Mitchell, 2006). However, for structural performance assessment, an explicit quantification of ground motion intensity, frequency content and duration is preferred, and therefore, the requirements from a duration metric for structures are different. An analysis and comparison of these duration metrics is presented in Chandramohan et al., 2013, which identifies the 5-95% significant duration ( $t_{5-95}$ ) as the metric best suited for use in hazard characterization and ground motion selection. This study therefore uses  $t_{5-95}$  to quantify ground motion duration as well.

## LONG AND SHORT DURATION SPECTRALLY EQUIVALENT SETS

As mentioned previously, the two biggest challenges that have faced researchers studying the effect of ground motion duration on structural response have been the scarcity of long duration ground motions and the difficulty in isolating the effect of ground motion duration from other ground motion characteristics, notably intensity and spectral shape. This study addresses these issues by employing two record sets, one containing long duration records, and the other containing spectrally equivalent short duration records.

The long duration set was first created by collecting ground motions recorded from the following large magnitude events: 1979 Imperial Valley (USA), 1985 Valparaiso (Chile), 2003 Hokkaido (Japan), 2008 Wenchuan (China), 2010 Maule (Chile) and 2011 Tohoku (Japan). Approximately 3700 horizontal record pairs were acquired and baseline corrected and filtered using the recommendations of Boore and Bommer, 2005 and Boore, 2005. Among these, long duration records were identified as those with  $t_{5-95}$  of at least one component  $> 45$ s. This criterion was not enforced on both components due to the limited number of available long duration records. Since  $t_{5-95}$  is a normalized metric, even low intensity ground motions with sufficiently long record lengths could have high values of  $t_{5-95} > 45$ s. Since the selected long duration ground motions were to be used for collapse analysis, all low intensity ground motions with mean  $PGA$  of both components  $< 0.1g$  or mean  $PGV$  of both components  $< 10\text{cm/s}$  were screened out. Finally, to prevent a single well-recorded event from dominating the record set, a maximum of 25 record pairs were retained from any single event. These restrictions resulted in 79 horizontal record pairs being available for the long duration set.

A short duration spectrally equivalent set was then created by matching each long duration ground motion to a corresponding ground motion with  $t_{5-95} < 45$ s from the PEER NGA West2 database (Ancheta et al., 2012) with a closely matching response spectral shape. In the matching procedure employed, the target response spectrum of the long duration ground motion was first sampled from 0.05s to 6s, at intervals of 0.05s to obtain 120 samples  $L_1, L_2, L_3, \dots, L_{120}$  with mean  $\bar{L}$ . The response spectrum of each ground motion from the database with  $t_{5-95} < 45$ s was also sampled at the same periods to obtain  $S_1, S_2, S_3, \dots, S_{120}$  with mean  $\bar{S}$ . The ground motion was then scaled using a factor  $k = \bar{L}/\bar{S}$  to make the sample mean of the scaled ground motion  $k\bar{S}$  equal to the sample mean of the long duration ground motion  $\bar{L}$ . A constraint of  $k \leq 5$  was placed to avoid the scaling of very low intensity records by large factors. The sum of squared errors  $SSE$ , used to quantify the error between the two response spectra was then computed as

$$SSE = \sum_{i=1}^{120} (L_i - kS_i)^2 \quad (1)$$

Among all the ground motions analyzed, the one with the lowest sum of squared errors was chosen as the best spectrally matching ground motion.

This short duration set was created as a control for the effect of spectral shape, so that any observed differences in the collapse capacities predicted by the two sets could be attributed to the differences in the durations of their ground motions. Figure 1 shows a comparison of the response spectra and time series of one long and short duration spectrally equivalent record pair. Figure 2 shows a comparison of the durations ( $t_{5-95}$ ) of ground motions in both sets.

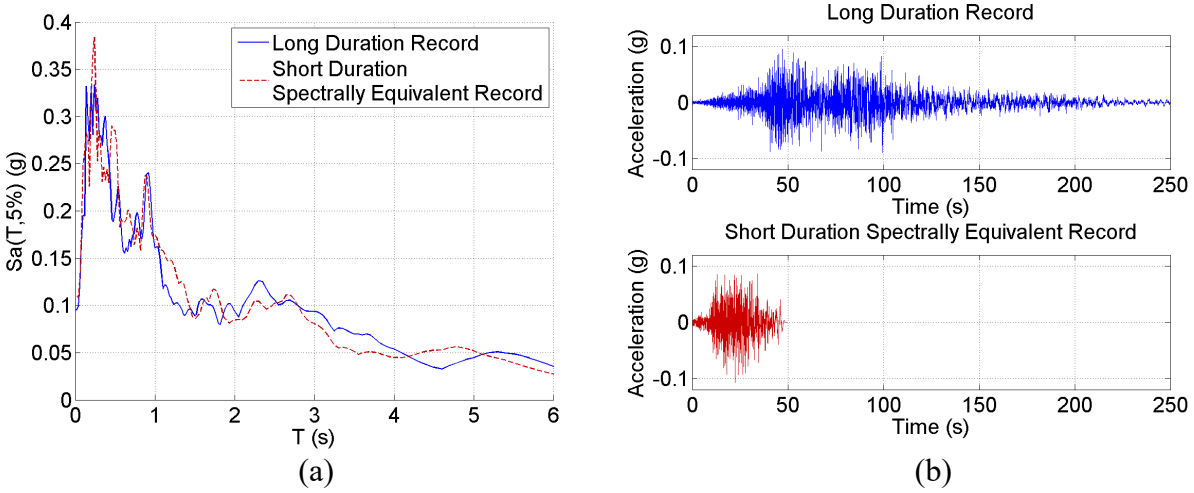


Figure 1. (a) Response spectra and (b) time series of a long and short duration spectrally equivalent record pair

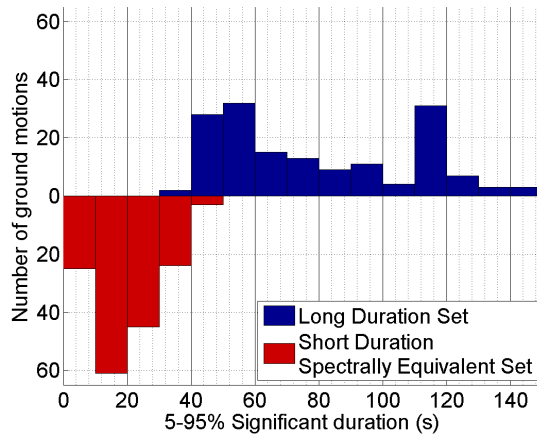


Figure 2. Durations ( $t_{5-95}$ ) of ground motions in the long and short duration spectrally equivalent sets

## BRIDGE PIER MODEL

The structure chosen to demonstrate the effect of ground motion duration was the reinforced concrete bridge pier tested by the Pacific Earthquake Engineering Research Center (PEER) and the George E. Brown Jr. Network for Earthquake Engineering Simulation (NEES) at the University of California, San Diego, as a part of the Concrete Column Blind Prediction Contest, 2010 ([http://nisee2.berkeley.edu/peer/prediction\\_contest](http://nisee2.berkeley.edu/peer/prediction_contest)). The structure was modeled in OpenSees, the Open System for Earthquake Engineering Simulation (McKenna et al., 2006). The column was modeled as a linear elastic beam-column element connected to the base using a zero-length rotational plastic hinge following the Modified Ibarra-Medina-Krawinkler peak-oriented hysteretic model (Ibarra et al., 2005). Figure 3 shows a schematic of the created model.

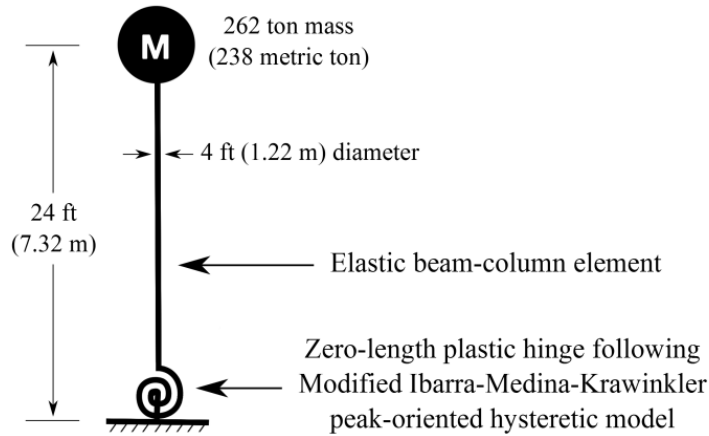


Figure 3. Bridge pier model schematic

The Modified Ibarra-Medina-Krawinkler peak-oriented hysteretic model includes a post-capping negative stiffness branch of the backbone curve to capture in-cycle deterioration, as well as an algorithm that cyclically deteriorates strength and stiffness based on the cumulative hysteretic energy dissipated. The parameters of the model were calibrated to the experimental measurements and the results of the calibration are shown in Figure 4. The period of the structure was found to be 1.2 s.

The model was then used to conduct incremental dynamic analysis (*IDA*) (Vamvatsikos and Cornell, 2002) using the long duration and spectrally equivalent short duration record sets, i.e. the same ground motion was scaled to different intensity levels until it caused structural collapse. The intensity of the ground motion when scaled to cause collapse represents the predicted collapse capacity of the structure. In each analysis, the engineering demand parameter (*EDP*) monitored was the peak chord rotation of the column, where a chord rotation threshold of 0.16 rad was used to indicate sidesway collapse. Figure 5 shows a plot of ground motion duration ( $t_{5-95}$ ) vs. predicted collapse capacity, and it can be observed that long duration ground motions on average cause collapse at lower intensities. The linear relation when plotted on log axes indicates a power law relation between  $t_{5-95}$  and predicted collapse capacity. In the case of this bridge pier model, the geometric mean collapse capacity predicted by the long duration set is 12% lower than that predicted by the short duration set. This percentage decrease in geometric mean collapse capacity predicted by the long duration set, compared to that predicted by the short duration set shall henceforth be referred to as  $\Delta_{dur}$ . Figure 6 shows the resulting *IDA* curves which demonstrate that the long duration ground motions on average produce a peak chord rotation of 0.077 rad just before collapse, compared to 0.097 rad produced by the short duration ground motions. This is believed to indicate that when large inelastic deformations occur, a long duration ground motion is more likely to lead to structural collapse by a combination of cyclic deterioration and ratcheting. Ratcheting is defined as the phenomenon by which lateral inelastic deformations that occur early in a response history lead to amplified  $P - \Delta$  moments that hasten subsequent sidesway collapse of the structure under later inelastic excursions in the same direction (Gupta and Krawinkler, 2000).

It may be noted here that the cyclic deterioration of the unloading stiffness commonly observed in reinforced concrete members could not be modeled in this study due to software limitations. Although the rules of a peak-oriented hysteretic model intrinsically incorporate post-yielding deterioration of the loading stiffness, the absence of unloading stiffness deterioration did not allow an accurate calibration of the model as observed in Figure 4. The observed results are thus expected to be conservative, and a larger effect of duration is expected if unloading stiffness deterioration is modeled.

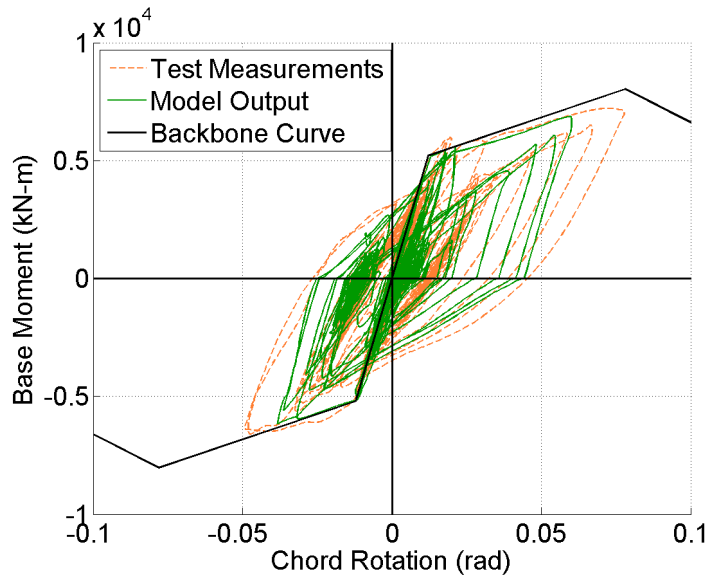


Figure 4. Calibration of bridge pier model to test measurements

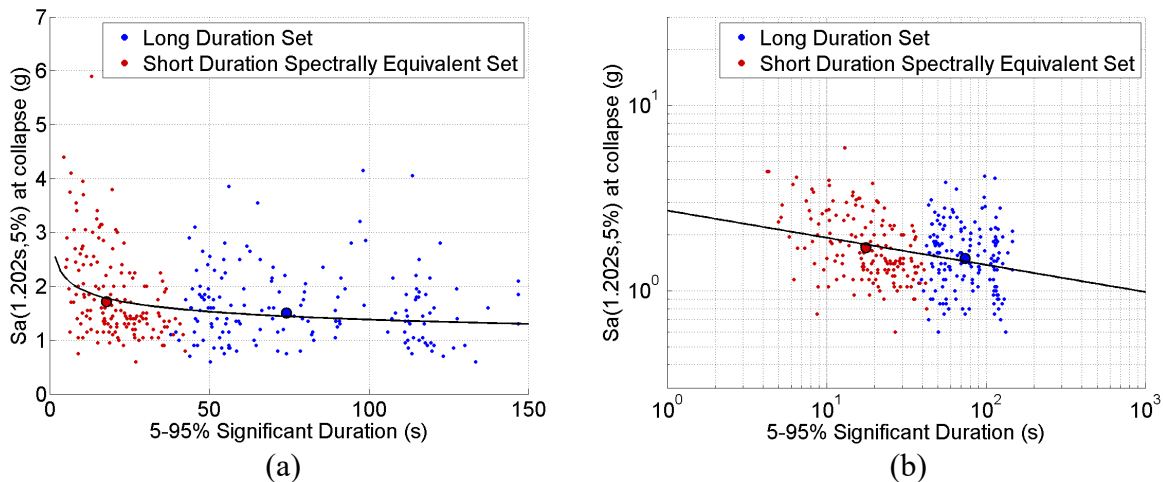


Figure 5. Collapse capacity vs. duration of base model on (a) linear axes and (b) log axes (Larger circles correspond to the geometric mean collapse capacity and geometric mean duration of all records in each set)

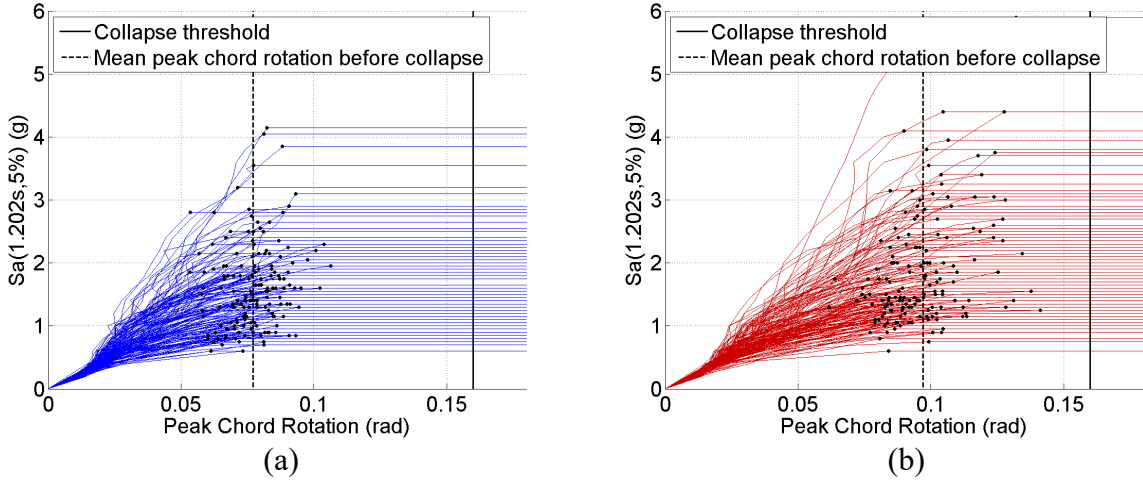


Figure 6. IDA curves of base model using (a) long duration and (b) spectrally equivalent short duration sets

## SENSITIVITY OF DURATION EFFECTS TO MODEL PARAMETERS

The results presented above are valid for the base model of a seismically designed and detailed reinforced concrete bridge pier, whose parameters were obtained by calibration to test measurements from a given loading. If the tested bridge pier were designed to have different dimensions, concrete mix, longitudinal and transverse reinforcement ratios, etc., the characteristics of the column would be altered significantly, and thereby so would the model parameters. Therefore, the sensitivity of the effect of ground motion duration quantified by  $\Delta_{dur}$ , to the model parameters requires examination. This study examines the sensitivity of  $\Delta_{dur}$  to two model parameters  $\gamma$  and  $\theta_p$  which are defined below.

The first parameter to be varied was  $\gamma$ , a dimensionless parameter used to control the rate of cyclic deterioration of the structure. The deterioration algorithm of the Modified Ibarra-Medina-Krawinkler hysteretic model first defines the reference hysteretic energy dissipation capacity of the column  $E_t$  as

$$E_t = \gamma M_y \theta_y \quad (2)$$

where  $M_y$  is the yield moment and  $\theta_y$  is the yield chord rotation of the column. Thereafter, the column's strength is deteriorated after every hysteretic excursion as

$$\beta_i = \left( \frac{E_i}{E_t - \sum_{j=1}^i E_j} \right)^c \quad (3)$$

$$F_i = (1 - \beta_i) F_{i-1} \quad (4)$$

where  $E_i$  is the hysteretic energy dissipated in the  $i^{th}$  excursion,  $F_i$  is the deteriorated strength after the  $i^{th}$  excursion, and  $c$  is an exponent commonly set to 1. The reader is referred to Ibarra et al., 2005 for a detailed description of the model. In summary, the larger the value of  $\gamma$ , the



larger the reference hysteretic energy dissipation capacity of the column, therefore slower the rate of deterioration, and vice versa. Figure 7(a) shows the variation of  $\Delta_{dur}$  with  $\gamma$  while all other model parameters are kept constant and indicates that the influence of duration diminishes as  $\gamma$  increases (rate of deterioration decreases). This is intuitively expected since in the absence of deterioration, the collapse capacity predicted by a long duration ground motion is not expected to differ by much from that predicted by a short duration ground motion. The maximum effect of duration is found to occur at  $\gamma = 20$ , where a 29% decrease in predicted geometric mean collapse capacity from the short duration set to the long duration set is observed. At values of  $\gamma$  lower than 20,  $\Delta_{dur}$  is found to decrease again since the extremely high rate of deterioration causes collapse to occur immediately after yielding as soon as any hysteretic energy is dissipated, thus nullifying the difference between a long duration and short duration ground motion. Figure 8 demonstrates the effect of deterioration on the hysteretic behavior of the column at collapse. Figures 8(a) and 8(b) show the hysteretic responses of the base model with  $\gamma = 120$  under typical short and long duration ground motions respectively at collapse. Greater deterioration is observed at collapse in the case of the long duration ground motion. Figure 8(c) shows the hysteretic response of the model with  $\gamma = 40$  under the same long duration ground motion as Figure 8(b) at collapse. The faster deterioration in the model with lower  $\gamma$  is evident.

The other parameter to be varied was  $\theta_p$ , the plastic rotational capacity of the column from the yield point to the capping point. The larger the value of  $\theta_p$ , the more ductile the structure. Figure 7(b) shows that  $\Delta_{dur}$  and hence the effect of duration increases with  $\theta_p$ . This again follows intuition since the more ductile a structure, the larger the number of inelastic deformation cycles it can sustain before collapsing, and hence the more it will deteriorate before collapsing.

The ranges over which the two parameters  $\gamma$  and  $\theta_p$  were varied in this study are consistent with the ranges of observed values of each parameter in the reinforced concrete column calibration database created by Haselton et al., 2008.

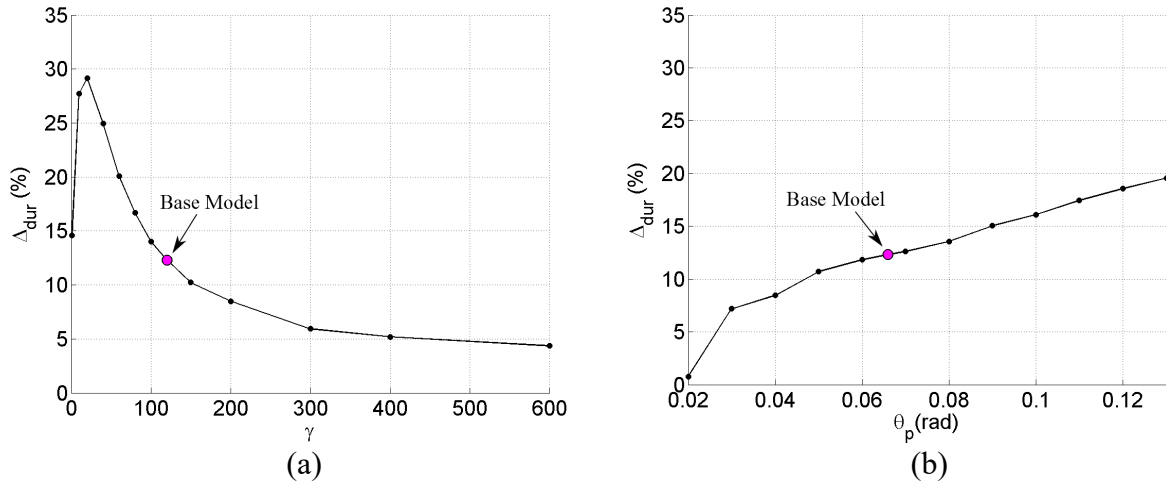


Figure 7. Sensitivity of  $\Delta_{dur}$  (percentage decrease in geometric mean collapse capacity predicted by the long duration set, compared to that predicted by the short duration spectrally equivalent set) to (a)  $\gamma$  (dimensionless parameter that controls rate of deterioration) and (b)  $\theta_p$  (plastic rotational capacity from yield to capping)

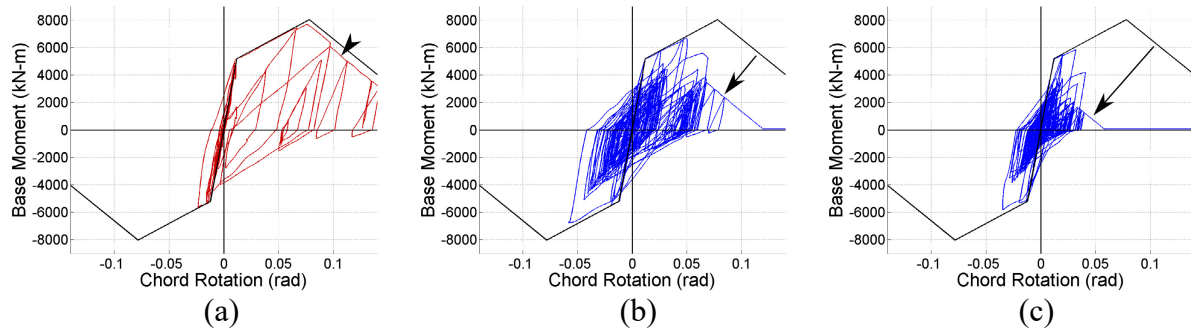


Figure 8. Hysteresis plots of (a) base model ( $\gamma = 120$ ) under short duration ground motion, (b) base model ( $\gamma = 120$ ) under long duration ground motion and (c) model with  $\gamma = 40$  under same long duration ground motion, at collapse

## CONCLUSION

This study verified that ground motion duration influences the predicted collapse capacity of a structure. This effect of duration could be isolated and quantified by employing spectrally equivalent long and short duration record sets. These record sets were designed such that each ground motion in the long duration set has a corresponding record in the short duration set possessing a similar spectral shape. Therefore any difference in a structure's collapse capacity as predicted by the two record sets could be attributed to the difference in the durations of their ground motions.

The magnitude of the effect of duration was found to depend on the characteristics of the structure. The reinforced concrete bridge pier model employed in this study demonstrated that the effect of duration is larger in structures that deteriorate rapidly and in moderately ductile structures. In the context of the concrete pier, this could be interpreted as the difference between the effect of duration on a concrete pier with modern ductile detailing (like the base model) and one with non-ductile detailing. The one with non-ductile detailing is expected to be less ductile and deteriorate more rapidly. Based on the observed trends, this would intuitively imply that the effect of decreasing both  $\gamma$  and  $\theta_p$  would simply cancel out. However in a separate analysis using  $\gamma = 40$  and  $\theta_p = 0.03$  rad (compare to  $\gamma = 120$  and  $\theta_p = 0.066$  rad used in the base model), the effect of decreasing  $\gamma$  was found to dominate and the value of  $\Delta_{dur}$  (percentage decrease in geometric mean collapse capacity predicted by the long duration set, compared to that predicted by the short duration spectrally equivalent set) corresponding to this set of model parameters was found to be 18%, which is greater than the 12% observed for the base model.

Values of  $\Delta_{dur}$  as high as 29% were observed by varying the model parameters within reasonable limits. A similar study on a 5-story steel special moment frame in Chandramohan et al. 2013 produced a  $\Delta_{dur}$  of 41% using the same record sets. It is thus concluded that it is important to take into account the durations of the ground motions used when estimating the collapse capacity of a structure, and not doing so could produce significant errors in the estimates.

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